Math 302
Spring 2020
Practice Exam 1

Name (Print): _____

Time Limit: 50 Minutes

Problem	Points	Score
1	13	
2	10	
3	15	
4	10	
5	20	
6	12	
7	20	
Total:	100	

No notes
No calculators
Simplify when possible
Show your work
Cheating will not be tolerated

Homework:
Classwork:
Current Grade:

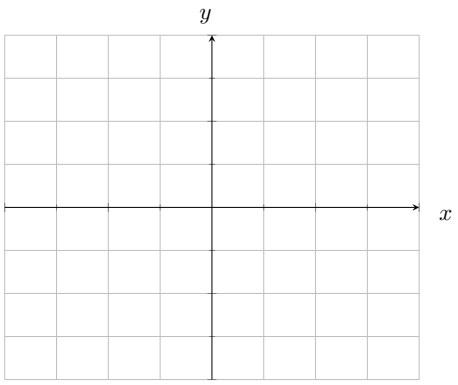
1. Consider the following differential equation:

$$\frac{dy}{dx} = \frac{y}{x-1}$$

(a) (2 points) Describe the isoclines of the direction/slope field.

(b) (2 points) Using the isoclines $y' = 0$, $y' = -1$ and $y' = -1$ populate the slope field below.

(c) (2 points) Graph an integral curve containing the point $(2, 2)$ on the slope field below



(d) (2 points) Graph an integral curve containing the point $(-1, -1)$

(e) (3 points) Solve the differential equation. Give precise equations of your integral curves from parts *c*) and *d*).

(f) (2 points) Identify any singular points.

2. (a) (8 points) Solve the following differential equation:

$$\frac{1}{x}\, dy = \sin(x)(y^3 + y)\, dx$$

(b) (2 points) Give a particular solution to the above differential equation that does not arise from any choice of constants.

3. (a) (3 points) Show that there is a function h such that $f(x,y) = x^n h(u)$ for $u = \dfrac{y}{x}$ if and only if there is a function k such that $f(x,y) = y^n k(\overline{u})$ with $\overline{u} = \dfrac{x}{y}$.

(b) (3 points) Show that the function is homogeneous and determine its order:

$$f(x,y) = \sqrt{x}y + \sqrt[6]{x^5}(x^2+y^2)^{\frac{1}{3}}$$

(c) (9 points) Given the differential equation

$$P(x,y)dx + Q(x,y)dy = 0,$$

prove that if $P(x,y)$ and $Q(x,y)$ are both homogeneous of the same order, then the substitution $u = \dfrac{x}{y}$ yields a separable differential equation.

4. (a) (8 points) Find a 1-parameter family of solutions

$$y^2\ dx + \left(x\sqrt{y^2 - x^2} - xy\right)\ dy = 0$$

(b) (2 points) Find a particular solution to the above that does not belong to the 1-parameter family of solutions.

5. (a) (10 points) Suppose that $\dfrac{\partial}{\partial y}P(x,y) = \dfrac{\partial}{\partial x}Q(x,y)$. If P and Q are sufficiently nice, find a function $f(x,y)$ such that $\dfrac{\partial}{\partial x}f(x,y) = P(x,y)$ and $\dfrac{\partial}{\partial y}f(x,y) = Q(x,y)$.

(b) (5 points) Show that the following differential equation is exact:

$$(4x^3 + y\sin(xy) + xy^2\cos(xy))\ dx + (4y^3 + x\sin(xy) + x^2y\cos(xy))\ dy = 0$$

(c) (5 points) Solve the following differential equation:

$$(4x^3 + y\sin(xy) + xy^2\cos(xy))\ dx + (4y^3 + x\sin(xy) + x^2y\cos(xy))\ dy = 0$$

6. (a) (2 points) Show that $y^2\ dx - x(x\ dy - y\ dx) = 0$ is not exact.

(b) (5 points) Find an integration factor for the above equation (hint: it's a function of xy^2).

(c) (5 points) Use the integration factor from part b) to solve the differential equation from part a).

7. (a) (5 points) If $y' + P(x)y = Q(x)$, find y.

(b) (5 points) Solve $y' - \cot(x)y = \sqrt{1 - \cos(x)}$

(c) (5 points) If $y' + P(x)y = Q(x)y^n$, find y.

(d) (5 points) Solve $xy' - y(2y\ln(x) - 1) = 0$